Dense Isocontour Imaging
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Abstract

We present an imaging technique intended to explore multi-scale image structures, represented by isophotes (lines of constant brightness) for photo images or in general case isolines. The cornerstone of the discussed technique is a view dependent periodic transfer function with the period depending on the gradient magnitude of the underlying scalar function such as to create a dense visualization independent of the gradient magnitude. We demonstrate that our approach is easy to implement, computationally efficient, and suitable for the fields that have reasonably structured isolines, i.e., that are sufficiently smooth.

CR Categories: I.4.10 [Computing Methodologies]: IMAGE PROCESSING AND COMPUTER VISION—Image Representation; I.3.3 [Computing Methodologies]: COMPUTER GRAPHICS—Picture/Image Generation;

Keywords: visualization, high dynamic range imaging

1 Introduction

In this work, we introduce a novel technique that is capable of displaying of as many isocontours (connected components of isolines) as possible at the given view scale without introducing any aliasing. This is done with a specifically designed transfer function, which reveals subtle variations in gradient orientation by the means of dense isocontour pattern. In order to achieve such a scale-agnostic representation, the original gradient magnitude has to be suppressed in the resulting image. This magnitude information, if required, can be easily shown with color coding on top of the isoline image. To the best of our knowledge, this type of image representation is difficult to impossible with other methods. Traditional techniques for geometric isocontour extraction such as marching cubes [Lorensen and Cline 1987] are suitable for revealing individual isoline components, however, they do not address the issue of optimal value sampling discussed below and filling the image with isolines. A related problem to the visualization of evolution of isolines can be found in the domain of high dynamic range imaging. One faces the same need to compress the image value range to be displayed on the low dynamic range devices. However while the HDR techniques such as tone mapping are commonly applied to real-world photo images which have natural restriction on a range of useful values and scales, our approach is suitable for various image sources, including non-photographic. For instance various simulation and acquisition techniques have larger variety of useful value ranges, making the compression desirable.

Also, unlike tone mapping, the isoline visualization admits adding high frequent details to the resulting image in order to enhance the geometry of the visualized lines, even if they aren’t present in the original data. In our technique we exploit a commonly recognized assumption [DiCarlo and Wandell 2000] that the human visual system is much more sensitive to local intensity ratio changes, corresponding to high spatial frequencies, than to global intensity differences, which is the basis of many tone mapping algorithms. This observation suggests that high-frequent sampling of the lines is preferable to perception. On the other hand, the exact representation of the isolines is not always possible, due to the Nyquist limit imposed by the finite resolution of the resulting image and the fractal structure of the lines. See for example the work of Khoury and Wenger [Khoury and Wenger 2010] for the thorough analysis of fractal dimension of the isosurfaces. This resolution dependency becomes an even more important factor when perspective projection on the screen is used, for example if the technique is applied on a surface in the 3D. In this case since different pixels on the screen correspond to different level-of-detail, some segments of the isolines should necessarily have coarser representation than the others. Consequently, the upper bound on the sampling frequency of the isolines in the screen space for the given view is space varying, which can be easily accounted for with our technique.

2 Technique Description

In this section we present an isoline generating transfer function with the following features:

• a uniform filling of the screen space;
• high isovalue sampling rate.

We achieve this goal, summarizing our considerations with Equation 4, which represents the multiscale harmonic transfer function with local spatial frequency normalization by he gradient magnitude. To give the rational behind this setup, we first have to explore the dense isoline imaging problem, starting with basic formal definitions.

Consider an image \( v(\bar{x}) \) defined on a domain \( \Omega \subset \mathbb{R}^2 \), under the assumptions that \( v \) is normalized such that \( 0 \leq v(\bar{x}) \leq 1 \) and the gradient magnitude \( |\nabla v| \) is bounded. The crucial aspect of the image representation is the spatial resolution, so we consider the isolines up to a certain level of detail in the following definition (Equation 1), motivated by the interval volumes concept known in visualization [Fujishiro et al. 1996]. We name a set of points \( S_{v_0,\epsilon} \) an isoline of the image \( v \) for a value \( v_0 \) up to the local error \( \epsilon \) if

\[
S_{v_0} = \{ \bar{x} \in \Omega : |v(\bar{x}) - v_0| < \epsilon(\bar{x}) \}
\] (1)

Given the definition 1, a naive design of a transfer function for sampling isolines periodically would be \( T(v) = \cos(2\pi v) \), corresponding to \( \epsilon = \pi \). However, uniform sampling in the image value space corresponds to varying spatial frequency of the resulting image \( T(v) \) (see Figure 1.b), causing aliasing in some regions and little or no details in the others. Our technique, explained further, consist in the generalization of this approach to control the spatial frequency of the transformed image.

For the proper visualization of the isolines essentially the perception of the normal direction to lines plays the crucial role. This information is transferred by the direction of gradient \( \nabla T \) of the visualization image \( T \), collinear to the gradient of the original data \( \nabla v \). In flow visualization where the direction is of particular importance, this is confirmed for example by some quality evaluation methods [Matvienko and Krüger 2012]. The orientation of \( \nabla v \) and the magnitude \( |\nabla v| \) are irrelevant and can be discarded. Since
\( \nabla T = T'(v) \nabla v \), virtually any function \( T(v) \) is suitable for representation of isolines in the aforementioned sense. The only points where \( T(v) \) fails to transfer the direction of the original gradient are those where \( T'(v) = 0 \). Additionally, a special care should be taken of the magnitude \( |\nabla T| \), which is related to the contrast of the resulting image and the resulting spatial frequency. We exploit the fact that the usefulness of the transfer function \( T(v) \) to isolate visualization is mainly determined by the action along the gradient in our further analysis. For the sake of simplicity we first consider the one-dimensional signal \( u(t) = v(x(t)) \), with critical points excluded from the domain, where \( \frac{du}{dt} = \nabla v \), i.e., the image along its gradient field, such that \( u'(t) = |\nabla v| \). We are going to use the notion of instantaneous frequency looking for a transfer function \( T(u, u') \) in the form \( T(u, u') = \cos(2\pi u g(u')) \) where the choice of function \( g(u) \) is to be clarified.

The instantaneous frequency \( f(t) \) for the signal \( u(t) = \cos(2\pi \phi(t)) \) is defined (see Boashash [Boashash 1992] for an overview) as the derivative of the instantaneous phase \( f(t) = \frac{d}{dt} \phi(t) \). Our goal is to design a band-limited transfer function \( T(u, u') = \cos(2\pi u g(u')) \) with instantaneous frequency \( \theta(t) = \frac{d}{dt} (u(t)g(u'(t))) = u'(t)g(u'(t)) + u(t)g'(u'(t))u''(t) \). Additionally a special care should be taken of the magnitude \( |\nabla T| \), virtually any function \( T(v) \) can be defined with \( b_1 - 1 \leq \log_2 (u') \leq b_1 \forall t \in \sigma_t \) and \( b_1 \) is a user-defined parameter, corresponding to the spacing between isolines. This corresponds to cutting the plot into pieces of unit height and shifting each piece to the desired frequency band, which is bounded by the Nyquist frequency at the top and the lowest acceptable frequency at the bottom. Intuitively, it is desirable to keep the length of segments \( \sigma_t \) small, but at the same time one should keep in mind that it should be larger than the wavelength \( \lambda \) to achieve full-period oscillation at the given frequency, so we have to impose a restriction on the growth of function \( \theta(t) = \log_2 u'(t) \).

Indeed, our method is applicable to the functions, satisfying the Lipschitz condition of order 1. That is, there exists a constant \( M \) such that for any pair of points \( t \) and \( t' \) Equation 2 holds:

\[
|\theta(t) - \theta(t')| \leq M |t - t'|
\]

Given that \( \max_{t,t' \in \sigma_t} |\theta(t) - \theta(t')| = 1 \), the upper bound for the choice of \( \lambda \) can be deduced easily: \( |t - t'| \geq \frac{1}{M} \geq \lambda \). From below \( \lambda \) is only bounded by the sampling frequency.

The implications of our method in 2D are illustrated by Figure 1.c and Figure 1.d. The definition of the transfer function in Equation 3...
is completely analogous to the one-dimensional case:

\[ T_i(v) = \cos \left( \frac{2\pi}{T} 2^{-b_i} v \right) \]  

(3)

The segments \( \sigma_i \subset \Omega \), defined as regions where \( b_i = \left\lceil \log_2 |\nabla v| \right\rceil \) is constant, feature prominent boundaries, illustrated by the Figure 1.c. Although the transfer function is generally discontinuous at the regions boundary, the lines of maximum brightness (white lines) are continuous. Indeed, looking closer at the behavior of the isolines at the boundary of adjacent regions \( \sigma_i \) and \( \sigma_j \) where \( b_j = b_i + 1 \) we immediately see that for \( T_i(v) \) attains maximum brightness on a isoline where \( v = 2^b \) since \( T_i(v) = T(0) = 1 \), while at the same time \( T_j(2v) = 1 \). In other words, every line from the segment \( \sigma_i \) is continued in the segment \( \sigma_j \) and additionally a new line is inserted in the middle between two existing lines. That is the line density is maintained automatically at a constant rate by introducing and terminating new lines (see Figure 1.b). The smoothness of the function can be further improved by adding lower-frequency harmonics with decreasing amplitude. Thus, the lines present on multiple scales will attain contrast, as a result of constructive interference between harmonics, while the discontinuities on the boundary will be smoothed with destructive interference. The resulting isoline-generating transfer function in its final form can be represented as a harmonic sum with exponentially decreasing weights for lower-frequency harmonics. (Equation 4):

\[ T_i(v, v') = \sum_{n=0}^{r} 2^{-n} \cos \left( \frac{2\pi}{\lambda} 2^{-\left(b_i+n\right)} v \right) \]

(4)

where the parameter \( r \) affects the smoothness/contrast of the resulting image. On the spectrogram the summation of the harmonics on different scales results in the overlap of frequencies in the time-frequency plane illustrated by Figure 2.b, corresponding to the harmonics overlap on the boundary regions in 2D, which produces visually appealing smooth transitions between regions. For example compare Figure 1.c, where one fixed frequency is used for the whole image to Figure 1.d, where several scales are combined. Also note that due to constructive interference, lines that persist on a large number of scales appear brighter.

3 Implementation and Results

In order to deliver a seamless demonstration our isocontouring technique and to show its computational efficiency, we have built a demo web-service, exploiting the experimental WebGL support of the latest versions of modern browsers. We created a HTML 5 and JavaScript application, running directly in browser, which allows to explore the effect of our function applied as an image filter and in a magic lens fashion on a set of real-world images. The function is computed in real-time in the shader, resulting in a highly interactive user interface. The demo is available at http://matvict.github.io/isocontouring/ To interact with the application we recommend using the latest versions of Google Chrome, as it provides the most complete and effective WebGL support. Application of our transfer function to real-world images apart from the artistic effect is useful for exploring the smooth gradient behaviors. For instance, its isocontour-enhancing property allows to visualize compression artifacts, arising from the block structure of the JPEG images (see Figure 4), which are barely noticeable in the color images.

4 Limitations and Future Work

We would like to mention that the use of our technique is only justified when there are some meaningful isolines to extract, that is the isolines are larger than the pixel size at the current viewing zoom. For noisy data such as CT scan images, the noise regions do not provide any useful information. Our transfer function is designed to discard gradient magnitude as much as possible to highlight isocountours independently of the local contrast. So far, serving to the advantage of our method, this feature turns out to be undesirable for images with significant amount of noise, because our technique immediately makes even low-contrast high-frequent noise prominent in the visualization. That is, the smoothness of the input data is an important requirement for successful use of our transfer function. To overcome this problem for the real-world photo-images images in our 2D demo we apply some Gaussian smoothing to achieve continuous isocountours. Also, the extension of the presented method to 3D fields and surfaces might be interesting. In particular the ability to control the spatial frequency in proportion to the local level of detail is a valuable feature for images resulting from perspective projection. See Figure 5 for the illustration of variable level of detail.

5 Conclusion

In this paper, we have explored the problem of dense isoline imaging, presenting a simple to use and effective technique based on a specially-designed transfer function. We demonstrated the artistic and smooth gradient-revealing effect of our method on photo images and generic scalar fields. We exploit its high run-time efficiency implementing it entirely in a web browser.

Acknowledgment

This research was made possible in part by the Intel Visual Computing Institute, and by the NIH/NCRR Center for Integrative Biomedical Computing, P41-RR12553-10 and by Award Number R01EB007688 from the National Institute Of Biomedical Imaging And Bioengineering. The content is under sole responsibility of the authors.
References


